Math 10A with Professor Stankova
Worksheet, Discussion #18; Friday, 10/6/2017
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Definite Integrals

Examples

1. Express $\lim_{n\to\infty}[e^{-1+3/n}\cdot\frac{3}{n}+e^{-1+6/n}\cdot\frac{3}{n}+\cdots+e^2\cdot\frac{3}{n}]$ as a definite integral on $[-1,2]$.

Solution: If we divide up the interval of $[-1,2]$ into n intervals, then each subinterval is of length is $\frac{2-(-1)}{n}=\frac{3}{n}$. Using right endpoints, the first term looks like $f(-1+3/n)\cdot\frac{3}{n}$ and hence we see that $f(x)=e^x$. Thus, this limit is

$$\int_{-1}^{2} e^x dx.$$

2. Express $\int_{-1}^{2}\cos(x)dx$ as a limit of right endpoint Riemann sums.

Solution: We split up the interval $[-1,2]$ into n intervals which are each of length $\frac{2-(-1)}{n}=\frac{3}{n}$. We start at -1 and end at 2 to get $[-1,-1+3/n],[-1+3/n,-1+6/n],\ldots,[2-3/n,2]$. Using the right endpoint method, we have that the integral is the limit

$$\lim_{n\to\infty}\left[\cos(-1+3/n)\cdot\frac{3}{n}+\cos(-1+6/n)\cdot\frac{3}{n}+\cdots+\cos(2)\cdot\frac{3}{n}\right].$$

3. True **FALSE** The addition definite integration law was proved using derivative laws.

4. **TRUE** False The addition definite integration law was proved using limit laws.

Problems

5. Express $\lim_{n\to\infty}[\tan(-1+2/n)\cdot\frac{2}{n}+\tan(-1+4/n)\cdot\frac{2}{n}+\cdots+\tan(1)\cdot\frac{2}{n}]$ as a definite integral from -1 to 1.

Solution: If we divide up the interval of $[-1, 1]$ into n intervals, then each subinterval is of length is $\frac{1-(-1)}{n} = \frac{2}{n}$. Using right endpoints, the first term is $\tan(-1 + 2/n) \cdot \frac{2}{n}$ and hence we see that $f(x) = \tan(x)$. Thus, this limit is

$$\int_{-1}^{1} \tan(x)dx.$$

6. Express $\lim_{n\to\infty}[\frac{2^3}{n^3} + \frac{2 \cdot 2^3}{n^3} + \cdots + \frac{2^3 n^2}{n^3}]$ as a definite integral from 0 to 2.

Solution: If we divide up the interval of $[0, 2]$ into n intervals, then each subinterval is of length is $\frac{2-0}{n} = \frac{2}{n}$. Using right endpoints, the first term is $(\frac{2}{n})^2 \cdot \frac{2}{n}$ and hence we see that $f(x) = x^2$. Thus, this limit is

$$\int_{0}^{2} x^2 dx.$$

7. Express $\lim_{n\to\infty}[(1 + \frac{3}{n} + \frac{9}{n^2}) \cdot \frac{3}{n} + (1 + \frac{6}{n} + \frac{36}{n^2}) \cdot \frac{3}{n} + \cdots + (1 + 3 + 3^2) \cdot \frac{3}{n}]$ as a definite integral from 0 to 3.

Solution: If we divide up the interval of $[0, 3]$ into n intervals, then each subinterval is of length is $\frac{3-0}{n} = \frac{3}{n}$. Using right endpoints, the first term is $(1 + \frac{3}{n} + \frac{9}{n^2}) \cdot \frac{3}{n} = (1 + \frac{3}{n} + (\frac{3}{n})^2) \cdot \frac{3}{n}$ and hence we see that $f(x) = 1 + x + x^2$. Thus, this limit is

$$\int_{0}^{3} 1 + x + x^2 dx.$$

8. Express $\int_{-1}^{3} \cos^2(x)dx$ as a limit of right endpoint Riemann sums.

Solution: We split up the interval $[-1, 3]$ into n intervals which are each of length $\frac{3-(-1)}{n} = \frac{4}{n}$. We start at -1 and end at 3 to get $[-1, -1 + 4/n], [-1 + 4/n, -1 + 8/n], \ldots, [3 - 4/n, 3]$. Using the right endpoint method, we have that the integral is the limit

$$\lim_{n\to\infty} \left[\cos^2(-1 + 4/n) \cdot \frac{4}{n} + \cos(-1 + 8/n) \cdot \frac{4}{n} + \cdots + \cos(3) \cdot \frac{4}{n}\right].$$

9. Express $\int_{-3}^{3} |x| dx$ as a limit of right endpoint Riemann sums.

Solution: We split up the interval $[-3, 3]$ into n intervals which are each of length $\frac{3-(-3)}{n} = \frac{6}{n}$. We start at -3 and end at 3 to get $[-3, -3 + 6/n], [-3 + 6/n, -3 + 12/n], \ldots, [3 - 6/n, 3]$. Using the right endpoint method, we have that the integral is the limit

$$\lim_{n\to\infty} \left[|-3 + 6/n| \cdot \frac{6}{n} + |-3 + 12/n| \cdot \frac{6}{n} + \cdots + |3| \cdot \frac{6}{n} \right].$$

10. Express $\int_{-2}^{0} |x^2 - x| dx$ as a limit of right endpoint Riemann sums.

Solution: We split up the interval $[-2, 0]$ into n intervals which are each of length $\frac{0-(-2)}{n} = \frac{2}{n}$. We start at -2 and end at 0 to get $[-2, -2 + 2/n], [-2 + 2/n, -2 + 4/n], \ldots, [-2/n, 0]$. Using the right endpoint method, we have that the integral is the limit

$$\lim_{n\to\infty} \left[|(-2 + 2/n)^2 - (-2 + 2/n)| \cdot \frac{2}{n} + |(-2 + 4/n)^2 - (-2 + 4/n)| \cdot \frac{2}{n} + \cdots + |0^2 - 0| \cdot \frac{2}{n} \right].$$

Fundamental Theorem of Calculus I

Examples

11. Evaluate the integral $\displaystyle\int_2^5 (x^2 + 1) dx$.

Solution: An antiderivative of $x^2 + 1$ is $\frac{x^3}{3} + x = F(x)$. So

$$\int_2^5 (x^2 + 1) dx = F(5) - F(2) = \frac{125}{3} + 5 - \frac{8}{3} - 2 = \frac{117}{3} + 3 = 39 + 3 = 42.$$

12. True **FALSE** Let $F(x)$ be defined on $[a, b]$ such that $F'(x) = f(x)$ on (a, b), then $\int_a^b f(x) dx = F(b) - F(a)$.

13. True **FALSE** $\int_{-1}^{1} \frac{1}{x} dx = \ln|x| \,|_{-1}^{1} = 1 - 1 = 0$.

14. True **FALSE** $\int f'(x) dx = f(x)$.

Problems

15. Evaluate the integral $\int_0^4 \sqrt{x}dx$.

Solution: We have that $\int_0^4 \sqrt{x}dx = \frac{2}{3} \cdot x^{3/2} \mid_0^4 = \frac{2}{3}(8-0) = \frac{16}{3}$.

16. Evaluate the integral $\int_1^8 \sqrt[3]{x}dx$.

Solution: We have

$$\int_1^8 \sqrt[3]{x} = \frac{3}{4}x^{4/3} \mid_1^8 = \frac{3}{4}(16-1) = \frac{45}{4}.$$

17. Evaluate the integral $\int_0^1 e^{x+1}dx$.

Solution: An antiderivative of e^{x+1} is itself so we can take

$$\int_0^1 e^{x+1}dx = e^{x+1} \mid_0^1 = e^2 - e^1 = e^2 - e.$$

18. Find the indefinite integral $\int (4t^3 + 3t^2)dt$.

Solution: The indefinite integral is

$$\int (4t^3 + 3t^2)dt = t^4 + t^3 + C.$$

19. Find the indefinite integral $\int \frac{1}{3x}dx$.

Solution: The indefinite integral is

$$\int \frac{1}{3x}dx = \frac{1}{3}\int \frac{1}{x}dx = \frac{\ln|x|}{3} + C.$$

20. Find the indefinite integral $\int 2\sin(2\theta)d\theta$.

Solution: The indefinite integral is

$$\int 2\sin(2\theta)d\theta = -\cos(2\theta) + C.$$